



## MODUL XI MATEMATIKA

<b>Judul</b>	<b>SISTEM PERSAMAAN LINEAR</b> (ringkasan awal, yang lengkap menyusul)	
<b>Penyusun</b>	<b>Distribusi</b>	<b>Perkuliahan</b>
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Tujuan :

Mahasiswa memahami pengertian dan dapat mencari matriks transpose, derterminan dan Invers Matriks

Materi:

1. Sistem persamaan Linear
2. Menyelesaikan SPL dengan Invers Matriks
3. Penyelesaian SPL Dengan Determinan
4. Penyelesaian SPL dengan eliminasi Gauss



3. Metode Eliminasi Langsung
4. Metoda determinan
5. Invers matriks  $AX=B \rightarrow X = A^{-1}.B$
6. Eliminasi Gauss-Jordan
7. Iterasi

**Contoh Soal :**

Tentukan himpunan jawab dari Sistem Persamaan Linier (SPL) :

$$2X_1 + 3X_2 + X_3 = 11$$

$$X_1 + X_2 + 2X_3 = 9$$

$$2X_1 + X_2 + X_3 = 7$$

Penyelesaian :

**(1) Dengan menggunakan bentuk matriks  $AX=B \rightarrow X = A^{-1}.B$**

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix} \longrightarrow \text{Diperoleh : } A^{-1} = \begin{pmatrix} -\frac{1}{6} & -\frac{2}{6} & \frac{5}{6} \\ \frac{3}{6} & 0 & -\frac{3}{6} \\ \frac{1}{6} & \frac{4}{6} & -\frac{1}{6} \end{pmatrix}$$

$$\begin{aligned} X = A^{-1} \cdot B &\longrightarrow \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{6} & -\frac{2}{6} & \frac{5}{6} \\ \frac{3}{6} & 0 & -\frac{3}{6} \\ -\frac{1}{6} & \frac{4}{6} & -\frac{1}{6} \end{pmatrix} \cdot \begin{pmatrix} 11 \\ 9 \\ 7 \end{pmatrix} = \\ &= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \end{aligned}$$

$$X_1 = 1, X_2 = 2, X_3 = 3$$

Himpunan Jawab (HJ) = {1, 2, 3}

Uji : Dengan memasukkan nilai  $X_1$ ,  $X_2$ , dan  $X_3$  ke Persamaan

**(2) Dengan menggunakan MATRIKS LENGKAP dan Operasi Baris Elementr:**

contoh :

$$\begin{pmatrix} 2 & 3 & 1 & 11 \\ 1 & 1 & 2 & 9 \\ 2 & 1 & 1 & 7 \end{pmatrix} \xrightarrow{b_{12}} \begin{pmatrix} 1 & 1 & 2 & 9 \\ 2 & 3 & 1 & 11 \\ 2 & 1 & 1 & 7 \end{pmatrix} \xrightarrow{\substack{b_{21}(-2) \\ b_{31}(-2)}} \begin{pmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -3 & -7 \\ 0 & -1 & -3 & -11 \end{pmatrix} \xrightarrow{\substack{b_{12}(-1) \\ b_{32}(1)}} \begin{pmatrix} 1 & 0 & 5 & 16 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & -6 & -18 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 5 & 16 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & -6 & -18 \end{pmatrix} \xrightarrow{b_3(-1/6)} \begin{pmatrix} 1 & 0 & 5 & 16 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & 1 & 3 \end{pmatrix} \xrightarrow{\substack{b_{13}(-5) \\ b_{23}(3)}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} \longrightarrow \begin{matrix} X_1 = 1 \\ X_2 = 2 \\ X_3 = 3 \end{matrix}$$

$$HJ = \{1, 2, 3\}$$

**Uji : Dengan memasukkan nilai  $X_1, X_2,$  dan  $X_3$  ke dalam Persamaan**

**(3) Penyelesaian SPL Dengan Determinan**

**SPL :**

$$\begin{matrix} a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n = b_1 \\ a_{21}X_1 + a_{22}X_2 + \dots + a_{2n}X_n = b_2 \\ \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \\ \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \\ a_{m1}X_1 + a_{m2}X_2 + \dots + a_{mn}X_n = b_m \end{matrix}$$

Ditulis ke dalam bentuk :

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \Lambda & a_{1n} \\ a_{21} & a_{22} & a_{23} & \Lambda & a_{2n} \\ M & M & M & O & M \\ a_{m1} & a_{m2} & a_{m3} & \Lambda & a_{mn} \end{vmatrix}$$

$$\Delta X_1 = \begin{array}{c} \left. \begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right\} \text{Koefisien ditukar dengan b} \\ \left| \begin{array}{ccccc} b_1 & a_{12} & a_{13} & \Lambda & a_{1n} \\ b_2 & a_{22} & a_{23} & \Lambda & a_{2n} \\ M & M & M & O & M \\ b_m & a_{m2} & a_{m3} & \Lambda & a_{mn} \end{array} \right| \end{array}$$

$$\Delta X_2 = \begin{array}{c} \left. \begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right\} \text{Koefisien ditukar dengan b} \\ \left| \begin{array}{ccccc} a_{11} & b_1 & a_{13} & \Lambda & a_{1n} \\ a_{21} & b_2 & a_{23} & \Lambda & a_{2n} \\ M & M & M & O & M \\ a_{m1} & b_m & a_{m3} & \Lambda & a_{mn} \end{array} \right| \end{array}$$

$$\Delta X_3 = \begin{array}{c} \left. \begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right\} \text{Koefisien ditukar dengan b} \\ \left| \begin{array}{ccccc} a_{11} & a_{12} & b_3 & \Lambda & a_{1n} \\ a_{21} & a_{22} & b_3 & \Lambda & a_{2n} \\ M & M & M & O & M \\ a_{m1} & a_{m2} & b_m & \Lambda & a_{mn} \end{array} \right| \end{array}$$

$$\Delta X_n = \begin{array}{c} \left. \begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right\} \text{Koefisien ditukar dengan b} \\ \left| \begin{array}{ccccc} a_{11} & a_{12} & a_{13} & \Lambda & b_1 \\ a_{21} & a_{22} & a_{23} & \Lambda & b_2 \\ M & M & M & O & M \\ a_{m1} & a_{m2} & a_{m3} & \Lambda & b_m \end{array} \right| \end{array}$$

Maka :

$$X_1 = \frac{\Delta X_1}{\Delta}$$

$$X_2 = \frac{\Delta X_2}{\Delta}$$

$$X_3 = \frac{\Delta X_3}{\Delta}$$

**Contoh Soal :**

Tentukan penyelesaian dari SPL :

$$3X_1 + 2X_2 - 2X_3 = 1$$

$$2X_1 - 3X_2 + X_3 = 1$$

$$5X_1 + X_2 - 3X_3 = 0$$

Jawab :

$$\Delta = \begin{vmatrix} 3 & 2 & -2 \\ 2 & -3 & 1 \\ 5 & 1 & -3 \end{vmatrix} = 27 + 10 - 4 - (30 + 3 - 12) \\ = 12$$

$$\Delta X_1 = \begin{vmatrix} 1 & 2 & -2 \\ 1 & -3 & 1 \\ 0 & 1 & -3 \end{vmatrix} = 9 - 2 - (1 - 6) \\ = 12$$

$$\Delta X_2 = \begin{vmatrix} 3 & 1 & -2 \\ 2 & 1 & 1 \\ 5 & 0 & -3 \end{vmatrix} = (-9 + 5 - 0) - (-10 + 0 - 6) \\ = 12$$

$$\Delta X_3 = \begin{vmatrix} 3 & 2 & 1 \\ 2 & -3 & 1 \\ 5 & 1 & 0 \end{vmatrix} = 10 + 2 - (-15 + 3) \\ = 24$$

$$X_1 = \frac{\Delta X_1}{\Delta} = \frac{12}{12} = 1 \quad ; \quad X_2 = \frac{\Delta X_2}{\Delta} = \frac{12}{12} = 1 \quad ; \quad X_3 = \frac{\Delta X_3}{\Delta} = \frac{24}{12} = 2$$

$$HJ = \{1, 1, 2\} \rightarrow X_1 = 1; X_2 = 1; X_3 = 2$$

**Uji : Dengan memasukkan nilai  $X_1$ ,  $X_2$ , dan  $X_3$  ke Persamaan**

**(4) Penyelesaian SPL dengan ESELON SPL → Eliminasi**

$$\begin{aligned} \text{SPL : } & a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ & a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ & \vdots \\ & a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{aligned}$$

Contoh 1 :

$$3X_1 + 2X_2 - 4X_3 = 9 \quad \dots (1)$$

$$2X_1 - 3X_2 + X_3 = 1 \quad \dots (2)$$

$$X_1 + X_2 + 3X_3 = 8 \quad \dots (3)$$

Tentukan bentuk Eselon SPL dengan Himpunan Jawab :  
Penyelesaian :

$$\begin{array}{r} (1-2) \\ 2x(1) \quad 6X_1 + 4X_2 - 8X_3 = 18 \\ 3x(2) \quad 6X_1 - 9X_2 + 3X_3 = 3 \\ \hline \quad \quad 13X_2 - 11X_3 = 15 \quad \dots (4) \end{array}$$

$$\begin{array}{r} (1-3) \\ 3x(3) \quad 3X_1 + 2X_2 - 4X_3 = 9 \\ \quad \quad 3X_1 + 3X_2 + 9X_3 = 24 \\ \hline \quad \quad -X_2 - 13X_3 = -15 \\ \quad \quad \quad X_2 + 13X_3 = 15 \quad \dots (5) \end{array}$$

$$\begin{array}{r} (4-5) \\ 13x(5) \quad 13X_2 - 11X_3 = 15 \\ \quad \quad 13X_2 + 169X_3 = 195 \\ \hline \quad \quad -180X_3 = -180 \\ \quad \quad \quad \underline{X_3 = 1} \end{array}$$

Sehingga :

$$\left. \begin{array}{l} 3X_1 + 2X_2 - 4X_3 = 9 \\ 13X_2 - 11X_3 = 15 \\ X_3 = 1 \end{array} \right\} \text{ BENTUK ESELON SPL}$$

$$\begin{array}{l} \hookrightarrow X_2 - 11 = 15 \\ \quad \quad 13X_2 = 26 \\ \quad \quad \underline{X_2 = 2} \end{array}$$

Substitusikan ke pers (1)

$$3X_1 + 4 - 4 = 9$$

$$\underline{X_1 = 3}$$

Himpunan Jawab : { 3, 2, 1 } → Uji : Dengan memasukkan nilai X<sub>1</sub>, X<sub>2</sub>, dan X<sub>3</sub> ke Pers

**Soal Latihan :**

$$5X_1 - 3X_2 + X_3 = 3$$

1.  $2X_1 + 2X_2 - 5X_3 = 1$

$$3X_1 + X_2 - 4X_3 = 0$$

$$X_1 - X_2 - X_3 = 0$$

2.  $-2X_1 + 3X_2 + 2X_3 = 1$

$$3X_1 - 4X_2 + 3X_3 = 5$$

$$2X_1 + 4X_2 - 5X_3 = 0$$

3.  $X_1 - X_2 - X_3 = 0$

$$3X_1 - 3X_2 - 2X_3 = 2$$

Selesaikan ketiga soal di atas masing – masing dengan :

- ◆  $X = A^{-1} \cdot B$
- ◆ Matriks Lengkap → Gunakan transformasi baris
- ◆ Dengan menggunakan Determinan
- ◆ Penyelesaian dengan Eselon SPL → Eliminasi

**Soal Tambahan**

Tentukan **Bentuk Eselon** dan **Himpunan Jawab** dari SPL

1.  $2X_1 + 3X_2 - 2X_3 + X_4 = 3$

$$3X_1 + 2X_2 + X_3 - 2X_4 = 7$$

$$4X_1 - 4X_2 - 3X_3 + 3X_4 = 1$$

$$X_1 + X_2 + 2X_3 - 3X_4 = 3$$

2.  $2X_1 + X_2 + X_3 - X_4 = 3$

$$3X_1 + 3X_2 - 2X_3 + 2X_4 = 11$$

$$4X_1 - 2X_2 - 2X_3 + 2X_4 = 2$$

$$5X_1 + 5X_2 + 3X_3 - X_4 = 0$$