1) $\begin{matrix} 2u+3y=1 \\ 3u+y=5 \end{matrix} \Rightarrow \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ $Ax=b. \Rightarrow x=A^{-1}b.$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{2-9}\begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 5 \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} 1-15 \\ -3+10 \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -14 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$HP = \{2, -1\}.$

2). $\begin{matrix} 3u+y=7 \\ 5u+2y=12 \end{matrix} \Rightarrow \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$

$$x = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 7 & 1 \\ 12 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix}} = \frac{(14-12)}{(6-5)} = \frac{2}{1} = 2.$$

$$y = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} 3 & 7 \\ 5 & 12 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix}} = \frac{36-35}{6-5} = \frac{1}{1} = 1.$$

$HP = [2, 1].$

3). $\begin{matrix} 2x-2y-2z=9 \\ x-6y-3z=-28 \\ 3x+2y+z=16 \end{matrix} \Rightarrow \begin{bmatrix} 2 & -2 & -2 \\ 1 & -6 & -3 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -28 \\ 16 \end{bmatrix}$

$$x = \frac{\begin{vmatrix} 9 & -2 & -2 \\ -28 & -6 & -3 \\ 16 & 2 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -2 & -2 \\ 1 & -6 & -3 \\ 3 & 2 & 1 \end{vmatrix}} = \frac{9\begin{vmatrix} -6 & -3 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} -28 & -3 \\ 16 & 1 \end{vmatrix} - 2\begin{vmatrix} -28 & -6 \\ 16 & 2 \end{vmatrix}}{2\begin{vmatrix} -6 & -3 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & -6 \\ 3 & 2 \end{vmatrix}} =$$

$$= \frac{9(0)+2(20)-2(40)}{2(0)+2(10)-2(20)} = \frac{40-80}{20-40} = \frac{-40}{-20} = 2.$$

$$y = \frac{\begin{vmatrix} 2 & 9 & -2 \\ 1 & -28 & -3 \\ 3 & 16 & 1 \end{vmatrix}}{-20} = \frac{2\begin{vmatrix} -28 & -3 \\ 16 & 1 \end{vmatrix} - 9\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & -28 \\ 3 & 16 \end{vmatrix}}{-20} =$$

$$= \frac{2(-28+48)-9(10)-2(16+84)}{-20} = \frac{40-90-200}{-20}$$

$$= 12,5.$$

$$z = \frac{\begin{vmatrix} 2 & -2 & 9 \\ 1 & -6 & -28 \\ 3 & 2 & 16 \end{vmatrix}}{-20} = \frac{2\begin{vmatrix} -6 & -28 \\ 2 & 16 \end{vmatrix} + 2\begin{vmatrix} 1 & -28 \\ 3 & 16 \end{vmatrix} + 9\begin{vmatrix} 1 & -6 \\ 3 & 2 \end{vmatrix}}{-20}$$

$$= \frac{2(-96+56) + 2(16+84) + 9(20)}{-20} = \frac{-80 + 200 + 180}{-20} = -15.$$

Bukti: $2x - 2y - 2z = 9$. HP: $[x, y, z] = [2, 12\frac{1}{2}, -15]$.

$2.2 - 2.12\frac{1}{2} + 30 = 4 - 25 + 30 = 9$. benar.

4). $u - 2y + z = 6$.
$3u + y - 2z = 4 \Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ 7 & -6 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 10 \end{bmatrix}$
$7u - 6y - z = 10$.

$A^{-1} = \frac{1}{\det(A)}(\text{adj } A)$

$C_{11} = +\begin{vmatrix} 1 & -2 \\ -6 & -1 \end{vmatrix} = -13$ $\quad C_{21} = -\begin{vmatrix} -2 & 1 \\ -6 & -1 \end{vmatrix} = -8$ $\quad C_{31} = +\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3$

$C_{12} = -\begin{vmatrix} 3 & -2 \\ 7 & -1 \end{vmatrix} = -11$ $\quad C_{22} = +\begin{vmatrix} 1 & 1 \\ 7 & -1 \end{vmatrix} = -8$ $\quad C_{32} = -\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = 5$

$C_{13} = +\begin{vmatrix} 3 & 1 \\ 7 & -6 \end{vmatrix} = -25$ $\quad C_{23} = -\begin{vmatrix} 1 & -2 \\ 7 & -6 \end{vmatrix} = -8$ $\quad C_{33} = +\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 7$.

$|A_1| = 1.-13 - 2.-11 + 25 = -13 + 22 - 25 = -16$.
$|A_2| = 3.-8 - 8 + 16 = -24 - 8 + 16 = -16$.
$|A_3| = 7.3 - 6.5 - 1.9 = 21 - 30 - 9 = -16$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -13 & -8 & 3 \\ -11 & -8 & 5 \\ -25 & -8 & 7 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \\ 10 \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -78 - 32 + 30 \\ -66 - 32 + 50 \\ -150 - 32 + 70 \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -80 \\ -48 \\ -112 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 7 \end{bmatrix}$$

Bukti:

$u - 2y + z = 6 \Rightarrow 5 - 6 + 7 = 6$ ✓.

5) $A = \begin{bmatrix} 2 & 3 & 4 & 0 \\ 7 & 5 & 0 & 0 \\ -3 & 4 & 12 & 0 \\ 4 & 6 & 8 & 0 \end{bmatrix}$ $\Rightarrow$ carilah invers.

$|A| = 0$, sesuai sifat det yaitu baris 1 & 4 kelipatan.
krn $|A| = 0$, matrik singular tdk punya det.

tt.