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CMC 101 TOPIK DALAM PEMROGRAMAN

PERTEMUAN 11

PROGRAM STUDI MAGISTER ILMU KOMPUTER

FAKULTAS ILMU KOMPUTER

TOPIK DALAM PEMROGRAMAN
Brute Force & Exhaustive Search

Pertemuan 11

TUJUAN PERKULIAHAN

- Mahasiswa memahami beberapa tipe persoalan yang penting.
- Selection Sort & Bubble Sort
- Sequential Search & Brute Force String Matching
- Closest Pair & Convex Hull dengan Brute Force
- Travelling Salesman Problem, Knapsack Problem, Assignment Problem

Brute Force & Exhaustive Search

- Straightforward way to solve a problem, based on the definition of the problem itself; often involves checking all possibilities
- Pros:
  - widely applicable
  - easy
  - good for small problem sizes
- Con:
  - often inefficient for large inputs

Brute Force Sorting

- Selection sort
  - scan array to find smallest element
  - scan array to find second smallest element
  - etc.
- Bubble sort
  - scan array, swapping out-of-order neighbors
  - continue until no swaps are needed
- Both take $\Theta(n^2)$ time in the worst case.

Brute Force Searching

- Sequential search:
  - go through the entire list of n items to find the desired item
- Takes Θ(n) time in the worst case

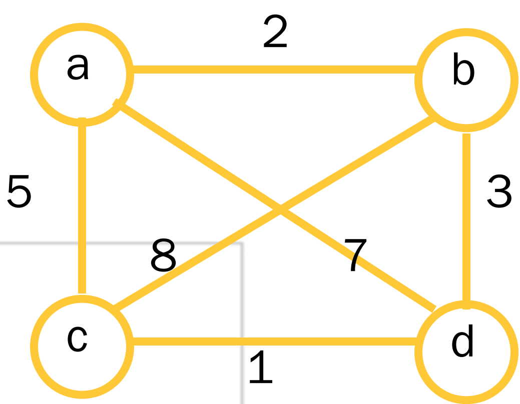
Brute Force Searching in a Graph

- (Review graph terminology and basic algorithms)
- Breadth-first search:
  - go level by level in the graph
- Depth-first search:
  - go as deep as you can, then backtrack
- Both take Θ(V+E) time, where |V| is the number of vertices and |E| is the number of edges

Brute Force for Combinatorial Problems

- Traveling Salesman Problem (TSP):
  - given a set of n cities and distances between all pairs of cities, determine order for traveling to every city exactly once and returning home with minimum total distance
- Solution: Compute distance for all “tours” and choose the shortest.
- Takes $\Theta(n!)$ time (terrible!)

TSP Example



a,b,c,d,a -> 18
a,b,d,c,a -> 11
a,c,b,d,a -> 23
a,d,b,c,a -> 23
a,d,c,b,a -> 18

Do we need to consider more tours?
Something odd about the “distances”?

TSP Applications

- transportation and logistics (school buses, meals on wheels, airplane schedules, etc.)
- drilling printed circuit boards
- analyzing crystal structure
- overhauling gas turbine engines
- clustering data

tsp.gatech.edu/apps/index.html
iris.gmu.edu/~khoffman/papers/trav_salesman.html

Brute Force for Combinatorial Problems

- Knapsack Problem:
  - There are n different items in a store
  - Item i weighs w_i pounds and is worth $\$v_i$
  - A thief breaks in
  - He can carry up to W pounds in his knapsack
  - What should he take to maximize his haul?

- Solution: Consider every possible subset of items, calculate total value and total weight and discard if more than W; then choose remaining subset with maximum total value.
- Takes $\Omega(2^n)$ time

Knapsack Applications

- Least wasteful way to use raw materials
- selecting capital investments and financial portfolios
- generating keys for the Merkle-Hellman cryptosystem

Knapsack Problems, H. Kellerer, U. Pferschy, D. Pisinger, Springer, 2004.

Knapsack Example

- item 1: 7 lbs, $42
- item 2: 3 lbs, $12
- item 3: 4 lbs, $40
- item 4: 5 lbs, $25
- W = 10

- need to check 16 possibilities

subset	total weight	total value
Ø	0	$0
{1}	7	$42
{2}	3	$12
{3}	4	$40
{4}	5	$25
{1,2}	10	$54
{1,3}	11	infeasible
{1,4}	12	infeasible
{2,3}	7	$52
{2,4}	8	$37
etc.		

Brute Force For Closest Pair

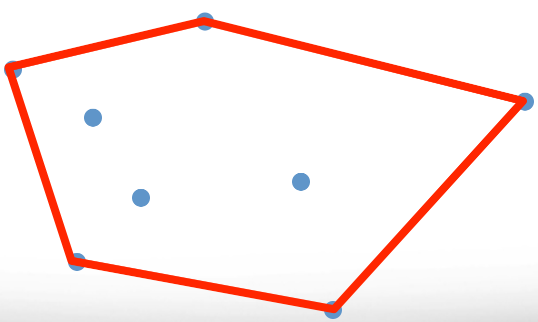
- Closest-Pair Problem:
  - Given n points in d-dimensional space, find the two that are closest
- Applications:
  - airplanes close to colliding
  - which post offices should be closed
  - which DNA sequences are most similar

Brute Force For Closest Pair

- Brute-force Solution (for 2-D case):
  - compute distances between all pairs of points
    - $\mathrm{sqrt}((x_i - x_j)^2 + (y_i - y_j)^2)$
  - scan all distances to find smallest
- Running time: $\Theta(n^2)$, assuming each numerical operation is constant time (including square root?)
- Improvements:
  - drop the square root
  - don't compute distance for same 2 points twice

Brute Force For Convex Hull

- Convex Hull Problem: Given a set of points in 2-D, find the smallest convex polygon s.t. each point in the set is enclosed by the polygon
  - polygon: sequence of line segments that ends where it begins
  - convex: all points on a line segment between 2 points in the polygon are also in the polygon



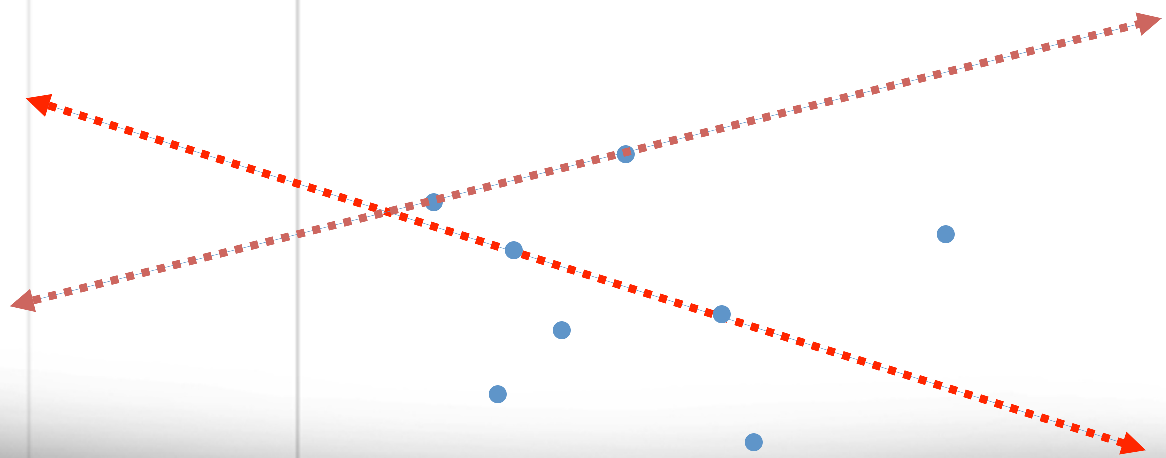
Convex Hull Applications

- In computer graphics or robot planning, a simple way to check that two (possibly complicated) objects are not colliding is to compute their convex hulls and then check if the hulls intersect
- Estimate size of geographic range of a species, based on observations (geocat.kew.org/about)

Brute Force For Convex Hull

- Key idea for solution: line passing through (x_i,y_i) and (x_j,y_j) is:

  $ax + by = c$ where $a = (y_j - y_i)$, $b = (x_i - x_j)$, $c = x_i\, y_j - y_i\, x_j$
- The 2 pts are on the convex hull iff all other pts are on same side of this line:



Brute Force For Convex Hull

- For each (distinct) pair of points in the set, compute a, b, and c to define the line $ax + by = c$.
  - For each other point, plug its x and y coordinates into the expression $ax + by - c$.
  - If they all have the same sign (all positive or all negative), then this pair of points is part of the convex hull.
- Takes $\Theta(n^3)$ time.

Brute Force for Two Numeric Problems

- Problem: Compute a^n
  - Solution: Multiply a by itself n-1 times
  - Takes Θ(n) time, assuming each multiplication takes constant time.
- Problem: multiply two nxn matrices A and B to create product matrix C
  - Solution: Follow the definition, which says the (i,j) entry of C is $\Sigma\, a_{ik} * b_{kj}$, k = 1 to n
  - Takes $\Theta(n^3)$ time, assuming each basic operation takes constant time

Brute Force/Exhaustive Search Summary

- sorting: selection sort, bubble sort
- searching: sequential search
- graphs: BFS, DFS
- combinatorial problems: check all possibilities for TSP and knapsack
- geometric: check all possibilities for closest pair and for convex hull
- numerical: follow definition to compute a^n or matrix multiplication

Applications of DFS

- Now let’s go more in depth on two applications of depth-first search
  - topological sort
  - finding strongly connected components of a graph

Depth-First Search

- Input: G = (V,E)
- for each vertex u in V do
  - mark u as unvisited
  - parent[u] := nil
- time := 0
- for each unvisited vertex u in V do
  - parent[u] := u // a root
  - call recursive DFS(u)

- recursiveDFS(u):
  - mark u as visited
  - time++
  - disc[u] := time
  - for each unvisited neighbor v of u do
    - parent[v] := u
    - call recursiveDFS(v)
  - time++
  - fin[u] := time

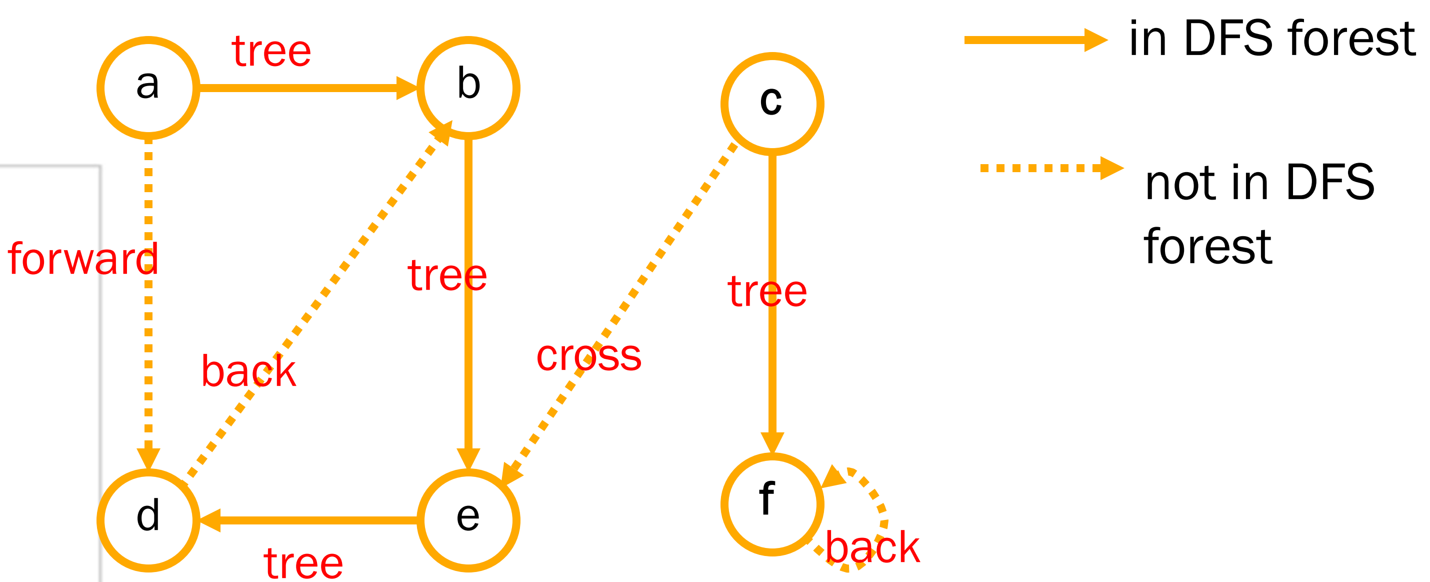
Nested Intervals

- Let interval for vertex v be [disc[v],fin[v]].
- Fact: For any two vertices, either one interval precedes the other or one is enclosed in the other.
  - because recursive calls are nested
- Corollary: v is a descendant of u in the DFS forest if and only if v's interval is inside u's interval.

Classifying Edges

- Consider edge (u,v) in directed graph G = (V,E) w.r.t. DFS forest
- tree edge: v is a child of u
- back edge: v is an ancestor of u
- forward edge: v is a descendant of u but not a child
- cross edge: none of the above

Example of Classifying Edges



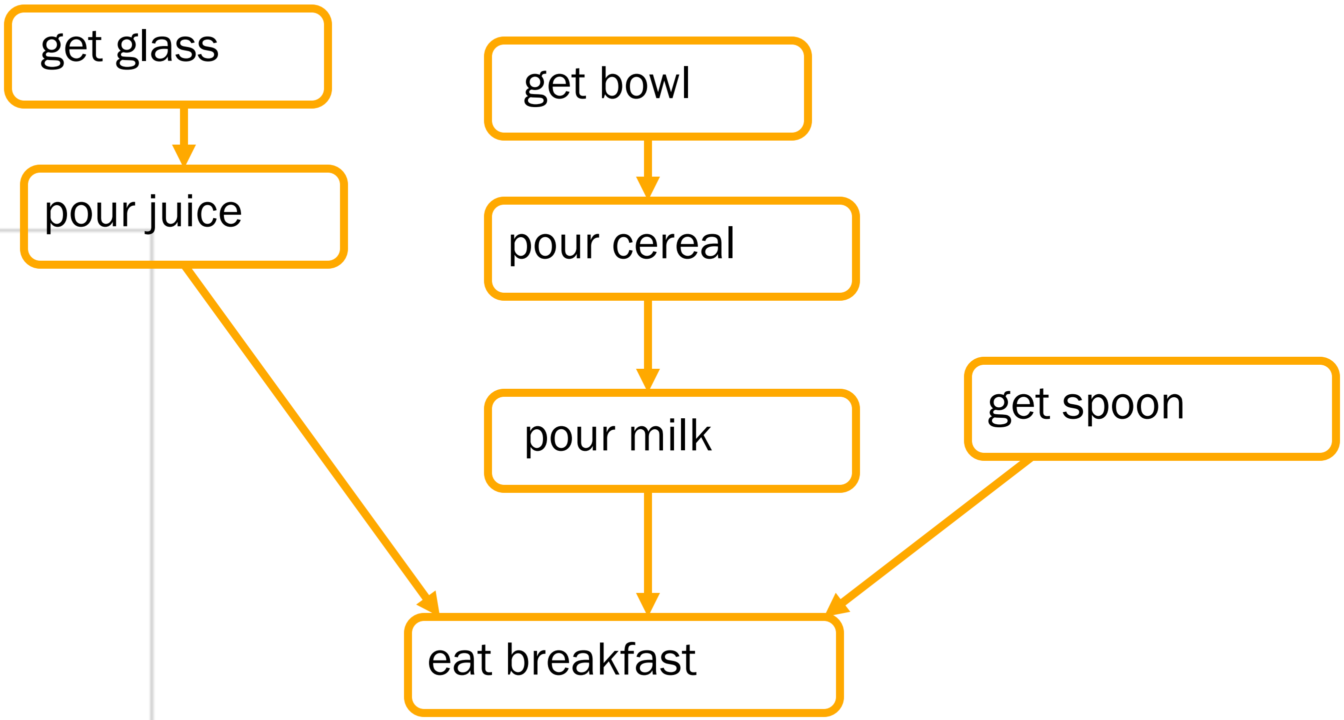
DFS Application: Topological Sort

- Given a directed acyclic graph (DAG), find a linear ordering of the vertices such that if (u,v) is an edge, then u precedes v.
- DAG indicates precedence among events:
  - events are graph vertices, edge from u to v means event u has precedence over event v
- Partial order because not all events have to be done in a certain order

Precedence Example

- Tasks that have to be done to eat breakfast:
  - get glass, pour juice, get bowl, pour cereal, pour milk, get spoon, eat.
- Certain events must happen in a certain order (ex: get bowl before pouring milk)
- For other events, it doesn't matter (ex: get bowl and get spoon)

Precedence Example



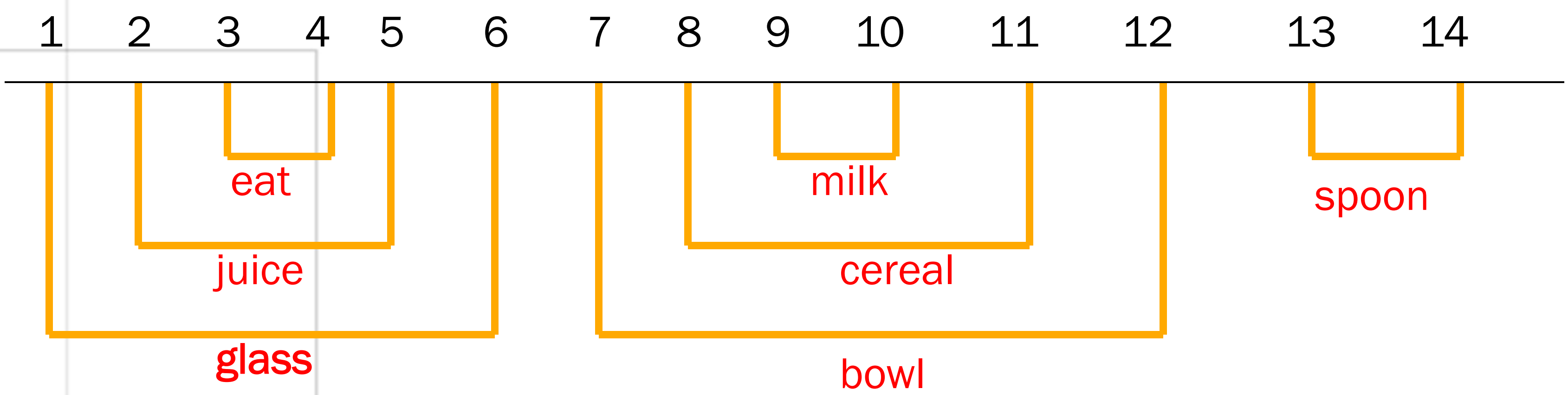
Order: glass, juice, bowl, cereal, milk, spoon, eat.

Why Acyclic?

- Why must directed graph by acyclic for the topological sort problem?
- Otherwise, no way to order events linearly without violating a precedence constraint.

Idea for Topological Sort Alg.

- Run DFS on the input graph



consider reverse order of finishing times:
spoon, bowl, cereal, milk, glass, juice, eat

Topological Sort Algorithm

input: DAG G = (V,E)

1. call DFS on G to compute finish[v] for all vertices v
2. when each vertex's recursive call finishes, insert it on the front of a linked list
3. return the linked list

Running Time: O(V+E)

Correctness of T.S. Algorithm

Show that if (u,v) is an edge, then v finishes before u finishes. Thus the algorithm correctly orders u before v.

u → v

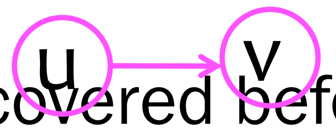
Case 1: u is discovered before v is discovered. By the way DFS works, u does not finish until v is discovered and v finishes.

disc(v) fin(v)
disc(u) fin(u)
v
u

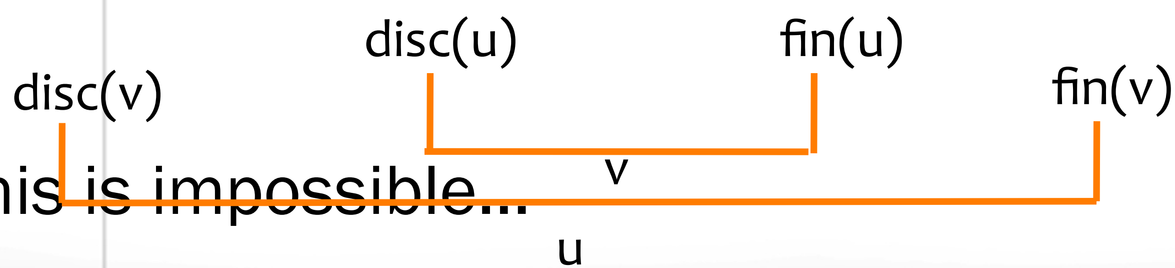
Then v finishes before u finishes.

Correctness of T.S. Algorithm

Show that if (u,v) is an edge, then v finishes before u finishes. Thus the algorithm correctly orders u before v.



Case 2: v is discovered before u is discovered. Suppose u finishes before v finishes (i.e., u is nested inside v).



Show this is impossible…

Correctness of T.S. Algorithm

- v is discovered but not yet finished when u is discovered.
- Then u is a descendant of v.
- But that would make (u,v) a back edge and a DAG cannot have a back edge (the back edge would form a cycle).
- Thus v finishes before u finishes.

DFS Application: Strongly Connected Components

- Consider a directed graph.
- A strongly connected component (SCC) of the graph is a maximal set of vertices with a (directed) path between every pair of vertices
- Problem: Find all the SCCs of the graph.

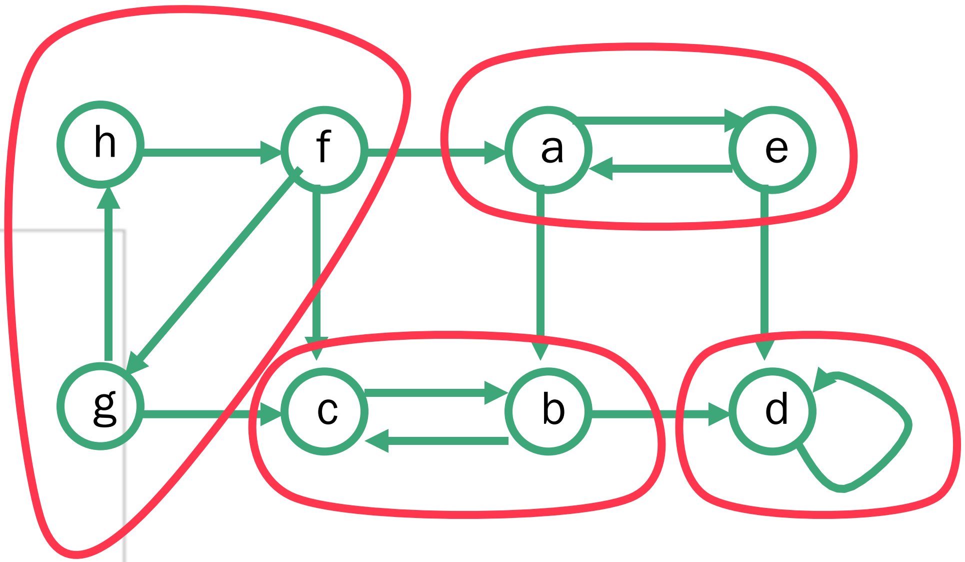
What Are SCCs Good For?

- Packaging software modules:
  - Construct directed graph of which modules call which other modules
  - A SCC is a set of mutually interacting modules
  - Pack together those in the same SCC

    www.cs.princeton.edu/courses/archive/fall07/cos226/lectures.html
- Solving the "2-satisfiability problem", which in turn is used to solve various geometric placement problems (graph labeling, VLSI design), as well as data clustering and scheduling

wikipedia

SCC Example



four SCCs

How Can DFS Help?

- Suppose we run DFS on the directed graph.
- All vertices in the same SCC are in the same DFS tree.
- But there might be several different SCCs in the same DFS tree.
  – Example: start DFS from vertex h in previous graph

Main Idea of SCC Algorithm

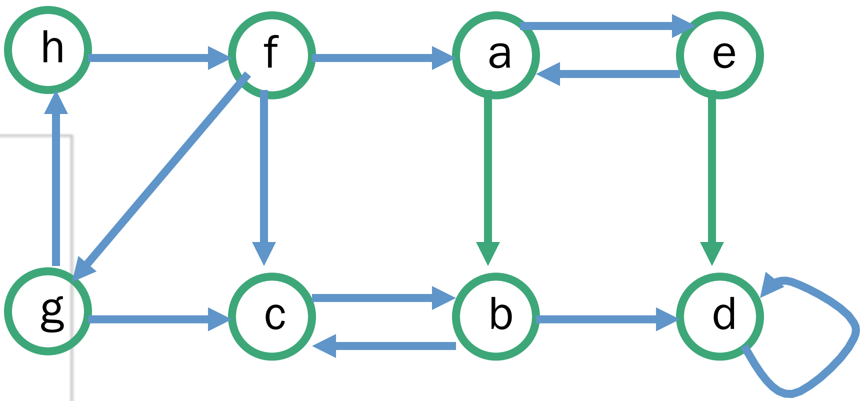
- DFS tells us which vertices are reachable from the roots of the individual trees
- Also need information in the "other direction": is the root reachable from its descendants?
- Run DFS again on the "transpose" graph (reverse the directions of the edges)

SCC Algorithm

input: directed graph G = (V,E)

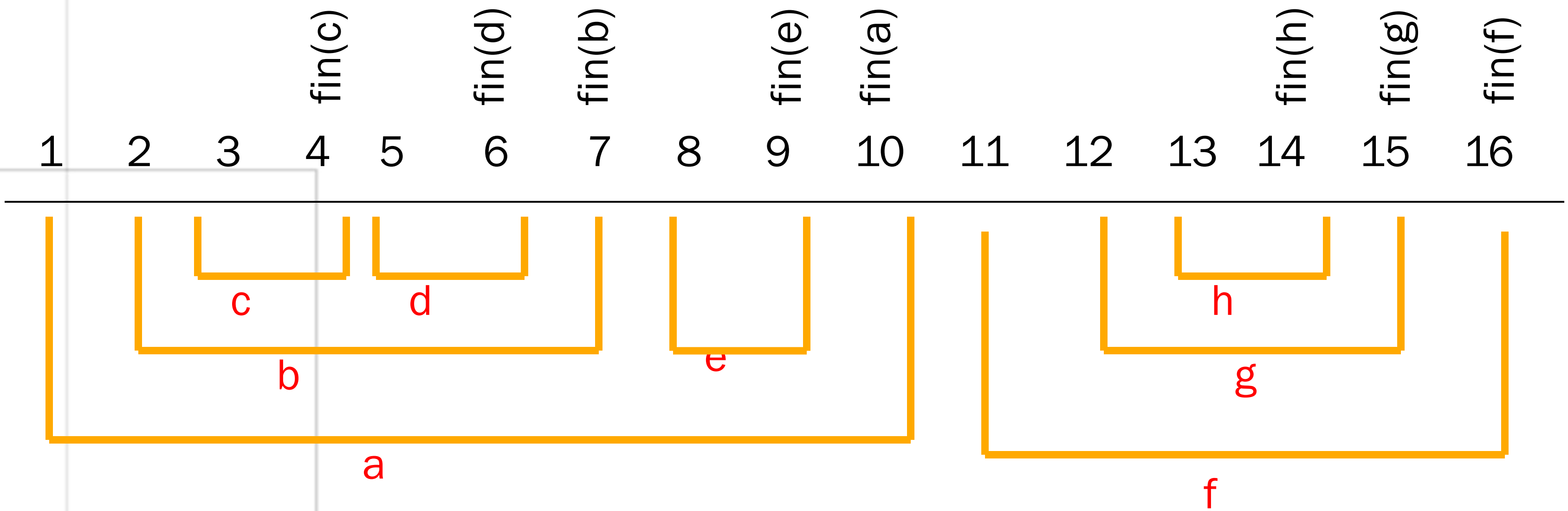
1. call DFS(G) to compute finishing times
2. compute G^T // transpose graph
3. call DFS(G^T), considering vertices in decreasing order of finishing times
4. each tree from Step 3 is a separate SCC of G

SCC Algorithm Example



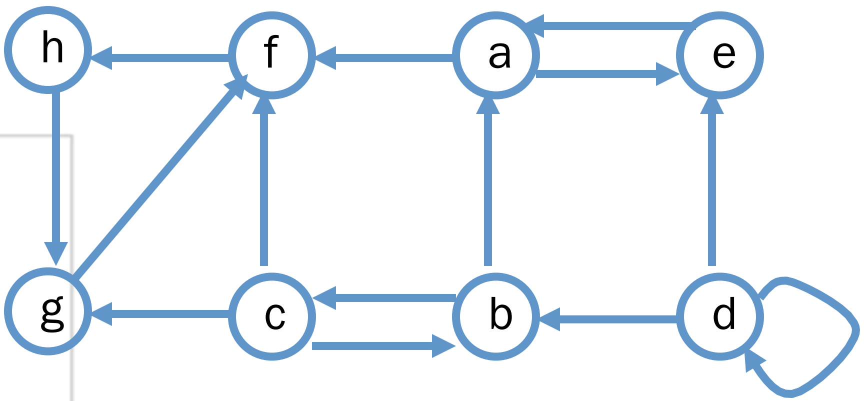
input graph - run DFS

After Step 1



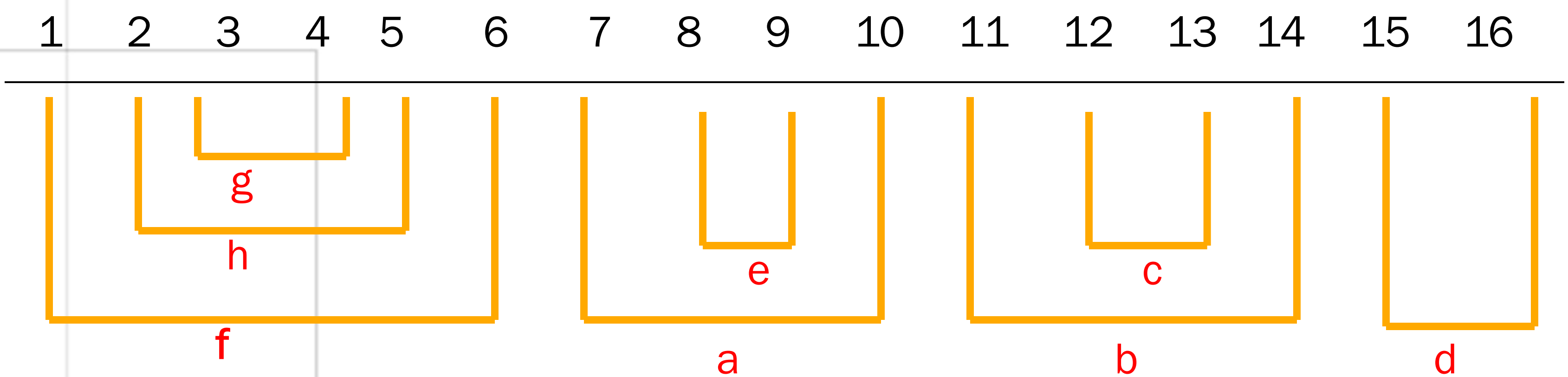
Order of vertices for Step 3: f, g, h, a, e, b, d, c

After Step 2



transposed input graph - run DFS with specified order of vertices

After Step 3



SCCs are {f,h,g} and {a,e} and {b,c} and {d}.

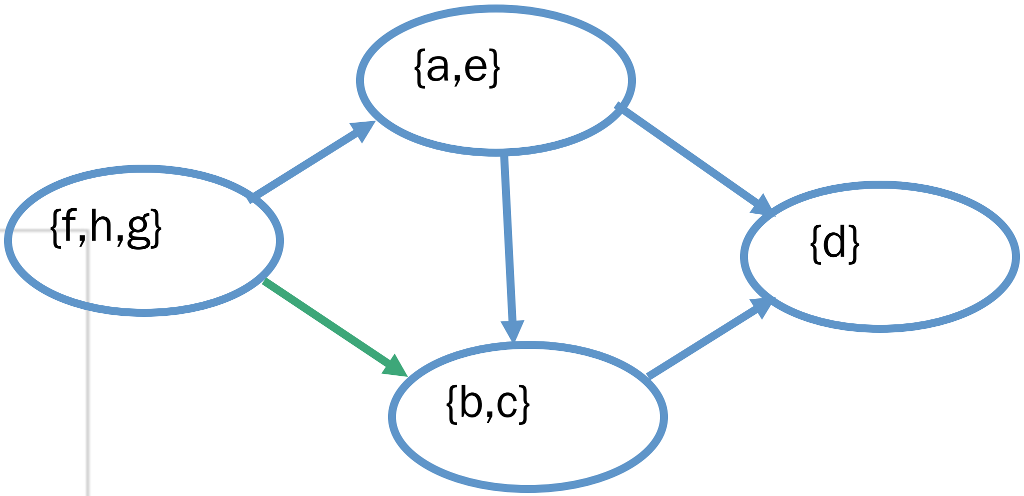
Running Time of SCC Algorithm

- Step 1: O(V+E) to run DFS
- Step 2: O(V+E) to construct transpose graph, assuming adjacency list rep.
- Step 3: O(V+E) to run DFS again
- Step 4: O(V) to output result
- Total: O(V+E)

Correctness of SCC Algorithm

- Proof uses concept of component graph, G^{SCC}, of G.
- Vertices are the SCCs of G; call them $C_1, C_2, \ldots, C_k$
- Put an edge from C_i to C_j iff G has an edge from a vertex in C_i to a vertex in C_j

Example of Component Graph



based on example graph from before

Facts About Component Graph

- Claim: G^{SCC} is a directed acyclic graph.
- Why?
- Suppose there is a cycle in G^{SCC} such that component C_i is reachable from component C_j and vice versa.
- Then C_i and C_j would not be separate SCCs.

Facts About Component Graph

- Consider any component C during Step 1 (running DFS on G)
- Let d(C) be *earliest* discovery time of any vertex in C
- Let f(C) be *latest* finishing time of any vertex in C
- Lemma: If there is an edge in G^{SCC} from component C' to component C, then

  $f(C') > f(C)$.

Proof of Lemma

- Case 1: d(C') < d(C).
- Suppose x is first vertex discovered in C'.
- By the way DFS works, all vertices in C' and C become descendants of x.
- Then x is last vertex in C' to finish and finishes after all vertices in C.
- Thus f(C') > f(C).

Proof of Lemma

- Case 2: d(C') > d(C).
- Suppose y is first vertex discovered in C.
- By the way DFS works, all vertices in C become descendants of y.
- Then y is last vertex in C to finish.
- Since C' → C, no vertex in C' is reachable from y, so y finishes before any vertex in C' is discovered.
- Thus f(C') > f(C).

SCC Algorithm is Correct

- Prove this theorem by induction on number of trees found in Step 3 (running DFS on G^T).
- Hypothesis is that the first k trees found constitute k SCCs of G.
- Basis: k = 0. No work to do !

SCC Algorithm is Correct

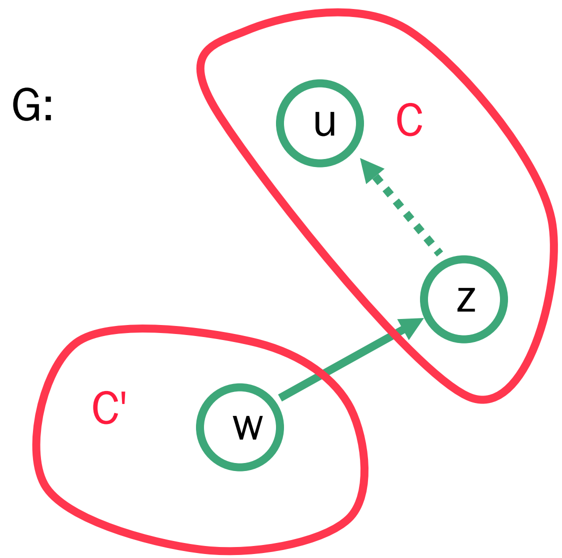
- Induction: Assume the first k trees constructed in Step 3 (running DFS on G^T) correspond to k SCCs; consider the (k+1)st tree.
- Let u be the root of the (k+1)st tree.
- u is part of some SCC, call it C.
- By the inductive hypothesis, C is not one of the k SCCs already found and all so vertices in C are unvisited when u is discovered.
  - By the way DFS works, all vertices in C become part of u's tree

SCC Algorithm is Correct

- Show *only* vertices in C become part of u's tree. Consider an outgoing edge from C.

SCC Algorithm is Correct

- By lemma, in Step 1 (running DFS on G) the last vertex in C' finishes after the last vertex in C finishes.
- Thus in Step 3 (running DFS on G^T), some vertex in C' is discovered before any vertex in C is discovered.
- Thus in Step 3, all of C', including w, is already visited before u's DFS tree starts



- Sumber : Prof. Jennifer Welch