

# The effect of individual neuromuscular properties on performance in sports

Sigrid Thaller\*, Markus Tilp and Martin Sust

Institut für Sportwissenschaft, Karl-Franzens-Universität Graz, A-8010 Graz, Austria

(Received 17 December 2009; final version received 24 March 2010)

We present a mathematical formulation of notions used in training science such as sports performance, performance determining factor and performance limiting factor. We give an example of model equations describing the relationship between individual neuromuscular properties and the associated performance in sports. An essential factor in modelling human movements is to determine the values of the subject's properties individually and in vivo. We perform measurements and identify the parameters describing the person's properties in the model equation. Simulations show effects of individual differences in the neuromuscular properties on the performance. Furthermore, we show the influence of changes in movement conditions on the performance.

Keywords: muscle; model; sport; performance

# 1. Introduction

In biomechanics, models based on Newton's fundamental equation of motion are widely used to describe human movement. Forces in movement equations should generally be formulated as force laws, that is, parameters in the force laws should be invariant and characterize constant properties of the object (e.g. the spring constant for mechanical springs) or movement conditions (e.g. temperature or air pressure). However, many models of human movement contain input quantities that are not movement-independent (e.g. torques as function of time) or they are a combination of conditions and properties (e.g. explosive force, start gradient, see [1]), thus leading to movement-specific results. Another shortcoming of most biomechanical models in the literature is the use of mean values for the subjects properties, possibly scaled to body dimensions as input parameters for the model equations [2,3]. It has been shown that individual muscle properties differ substantially from mean values even in a homogeneous group of subjects [4]. Thus, simulations lead to mean results which are not subject-specific. Therefore, obtaining individual values for the input parameters of the model equations is a crucial challenge but essential for predicting subject-specific movements.

During human movements, active forces are developed by the contraction of muscles. A classical and accepted model for the contraction of skeletal muscle is the Hill-type model that characterizes the (concentric) force–velocity relationship of a muscle [5–7]. To use this muscle model for realistic human movements or to determine model parameters individually, specific structural conditions have to be considered. Conditions that allow combining different muscle fibres were investigated in [8,9]. For single muscles, muscle properties have

<sup>\*</sup>Corresponding author. Email: sigrid.thaller@uni-graz.at

been determined by different methods, see, for example, [10–12]. Conditions that allow combining several muscles to model muscles have been investigated in [9,12]. The conditions under which the activation can be formulated such that the forces still are a force law have been studied in [13]. Conditions on the structure of force laws for muscle forces are formulated in [8].

The article is organized as follows: in Section 2 we establish conditions for modelling in sports science and present a mathematical formulation for notions in sports science such as performance or performance determining factors. We define the performance function that gives information on the relation between the persons properties and the performance in sports and we state some facts on the force law for the muscle. Section 3 briefly describes the model and the kind of neuromuscular properties we want to determine. Section 4 shows the results of measurements and the associated parameter estimation. In the simulation section (Section 5), we show the effect of interindividual differences and the application of modelling in sports science. The last section concludes the article and summarizes the most important facts.

### 2. Theoretical considerations on a model-based training science

Let *n* be the number of parameters in a set of model equations describing a human movement. The parameters can be divided into two disjoint classes: parameters that describe the person's properties (such as leg length, activation rate, mass, muscle properties) and parameters that describe the movement conditions (such as the mass of a pushed object, the room temperature, the initial condition of movement). Let k < n be the number of parameters describing the movement conditions and m < n the number of the person's properties, n = k + m.

**Remark 1:** The property values of a person are independent of the conditions of the considered movement. Note in particular, that a person's property value is the same for all movements, that is, it does not depend on the movement.

## 2.1. Performance function

In many sports, the performance of the athlete is assessed by a real number, for example, the length of a jump or the running time in sprint. If the model of the movement contains all the relevant information of the movement, the performance can be calculated using the parameters of the model equations.

**Definition 2.1:** A performance of a modelled movement is a real number *z* which can be calculated using the parameters occurring in the model equations.

**Remark 2:** There are many possible performances that can be defined for a human movement. Not all make sense in sports science. Therefore, we will restrict our considerations on performances that can be interpreted in sports science, such as maximum velocity of a movement, mean velocity, jump height or time.

**Remark 3:** If the performance is calculated using a real subset of the parameters of the model equation, the other parameters do not influence the performance and therefore are not necessary for describing the performance. This shows that the model is not as simple as possible and as complex as necessary, what is expected of a good model.

**Definition 2.2:** The performance function f is a function between the set D of properties and a one-dimensional performance space, defined by

$$\begin{aligned} f: D \subset \mathbb{R}^m \to \mathbb{R} \\ x \in D \mapsto z \in R, \end{aligned} \tag{1}$$

where x is the state of the person, an m-tuple of relevant properties. The graph of this mapping is a surface showing the relation between the properties and the performance.

**Remark 4:** The performance function depends on the fixed values of the conditions of the movement. Of course, it would also be possible to define the performance and performance function including the conditions. The reason for holding the conditions fixed is that in most applications in sports science the properties are changed by training, whereas most conditions are fixed due to regulations. This reduces the dimension of the domain of the function and therefore simplifies the applications. Further reduction can be obtained regarding the restriction of f to some subspace of  $\mathbb{R}^m$ , setting some of the property values fixed. In specific situations, however, it will be appropriate to include at least some conditions.

**Remark 5:** The set D is a real subset of  $\mathbb{R}^m$ , because not all combinations of property values occur in reality. The set D is bounded and open. For example, a person with long legs in general has more weight, so the property values have to fulfil certain statistical relations (see, for example [4]). We will meet another example in the Subsection 2.2 dealing with parameters in Hill's equation.

**Definition 2.3:** Let  $E_1, ..., E_m$  be the properties of the person and consider a state x of the person.

- (a)  $E_i$  is called performance determining factor, if for fixed values of  $E_j$ ,  $j \neq i$ , a change in the value of  $E_i$  changes the performance z.
- (b) E<sub>i</sub> is called performance limiting factor, if any variation in the value of E<sub>j</sub>, j ≠ i, E<sub>i</sub> fixed, does not change the performance z.

**Remark 6:** An equivalent formulation of the definition of the performance limiting factor is the following: Given a state x. Only a change in the value of  $E_i$  leads to a change in performance.

# 2.2. Some considerations on the force law for muscles

Hill's equation [4,13,14] on the force-velocity relation of a muscle, given by

$$f = \frac{c}{v+b} - a,\tag{2}$$

contains a lot of information relevant for sports. Here f denotes the concentric contraction force of the muscle, v is the contraction velocity of the muscle, and a, b and c are a set of parameters greater than 0, describing the muscle properties of the person.

**Remark 7:** Because of the interpretation of the variables, *f* and *v* have to be positive and the function is convex in the considered interval  $0 \le v \le c/a - b = v_{\text{max}}$ .

**Remark 8:** Hill's equation describes the force as a function of velocity in case the muscle is fully activated. If the activation is lower, that is, less motor units that could be activated at the same time are activated at this moment, the muscle force also is lower.

**Remark 9:** The mechanical power *p* of the muscle is given by the product of force *f* with velocity v, p = fv. Evaluating the derivation dp/dv leads to the optimum velocity  $v_{opt}$ , defined as the velocity at which the muscle can exert the maximum power. This yields

$$v_{\rm opt} = -b + \sqrt{\frac{cb}{a}} \tag{3}$$

and

$$f_{\rm opt} = -a + \sqrt{\frac{ca}{b}},\tag{4}$$

 $f_{\rm opt}$  denoting the corresponding optimum force.

**Remark 10:** The set of parameters *a*, *b* and *c* in Hill's equation can be replaced by an equivalent set of parameters,  $f_{\text{max}}$ ,  $v_{\text{max}}$  and  $p_{\text{max}}$ , denoting the isometric force, the maximum possible velocity and the maximum possible mechanical power of the muscle. These parameters are well known by sports scientists and coaches. We have

$$f_{\max} = \frac{c}{b} - a \tag{5}$$

and

$$v_{\max} = \frac{c}{a} - b \tag{6}$$

$$p_{\max} = ab + c - 2\sqrt{abc} \,. \tag{7}$$

The shape of Hill's force-velocity relation provides information about the endurance of the muscle. The curvature can be measured by the ratio of  $a/f_{\text{max}}$ . Endurance athletes and beginners have more curved force-velocity relations ( $a/f_{\text{max}} \le 0.30$ ) than athletes in power sports ( $a/f_{\text{max}} \ge 0.30$ ) [1]. Another relationship with the curvature can be found in the efficiency, defined as ratio  $p_{\text{max}}/c$ . The distribution between slow-and fast-twitch fibres in the muscle is related to  $b_n$ , the value of b normalized to the muscle length n. Larger values of  $b_n$  correlate with a higher percentage of fast-twitch fibres [4,9].

An important question in sports science is the difference between individuals and the possible consequences on the movement. Given two different sets of parameters values,  $a_1$ ,  $b_1$ ,  $c_1$ , and  $a_2$ ,  $b_2$ ,  $c_2$ , we can distinguish between three cases (see Figure 1):

(1) The force-velocity relations do not intersect, one muscle has more force than the other for all contraction velocities.



Figure 1. Force-velocity relations with no intersection (*left*), one point of intersection (*middle*) and two points of intersection (*right*).

(2) There is just one point of intersection. If this point is not an osculation point, one muscle can exert more force at slower contraction velocities, whereas the other one has more force when the velocity increases. The condition for getting only one intersection point which is not an osculation point is

$$(v_{\max,1} - v_{\max,2})(f_{\max,1} - f_{\max,2}) < 0.$$
(8)

(3) There are two points of intersection. In this case, the curvature of the functions has to be different. The range of velocity near the optimal velocity leads to more force for athletes in power sports.

**Remark 11:** Note that we only look at intersection points  $(v_{int}, f_{int})$  in the first quadrant of the (v, f)-plane, that is, we always have the constraint  $0 \le v_{int} \le v_{max}$ .

The intersection points can be calculated explicitly by setting

$$f = \frac{c_1}{v + b_1} - a_1 = \frac{c_2}{v + b_2} - a_2,$$
  
$$v_{\text{int}} = \frac{1}{2} \cdot \left(\frac{c_1 - c_2}{a_1 - a_2} - (b_1 + b_2)\right),$$
(9)

for an osculation point, and

leading to

$$v_{\text{int}} = \frac{1}{2} \cdot \left(\frac{c_1 - c_2}{a_1 - a_2} - (b_1 + b_2)\right) \pm \sqrt{\frac{1}{4} \cdot \left(\frac{c_1 - c_2}{a_1 - a_2} - (b_1 + b_2)\right)^2 - \frac{c_1 b_2 - c_2 b_1}{a_2 - a_1} - b_1 b_2},$$
(10)

in all other cases, provided  $0 \le v_{\text{int}} \le v_{\text{max}}$ .

**Remark 12:** Equations (9) and (10) for the intersection points can also be expressed using the parameters  $f_{\text{max}}$ ,  $v_{\text{max}}$  and  $p_{\text{max}}$ , as well as the condition for just one intersection point, Equation (8), might be formulated using the parameters a, b and c.

**Remark 13:** Equation (8) is independent of  $p_{\text{max}}$ , that is, it is independent of the curvature of the force-velocity relation and therefore independent of the efficiency of the athlete.

#### 3. Model of a human movement

We want to exemplify the benefit of modelling in sports science by the following simple model of a knee extension movement. Consider a leg extension on an inclined leg press with inclination angle  $\alpha$ , where a subject pushes a mass *m* under maximum voluntary contraction (MVC).

To describe this movement, we use a model for the extension movement with a hinge joint (see, for example [12]). The extensor muscles are described by a model muscle. The force–velocity relation of this muscle is given by Hill's equation as defined in the last section. The activation process of the muscle under maximum voluntary contraction is described by a time-dependent function S:

$$S(t) = 1 - \exp(-A(t - t_0) + 1 - \exp(-A(t - t_0))).$$
(11)

**Remark 1:** The function *S* ranges between 0 and 1. S(t) = 0 describes the situation that the muscle is not activated at all at time *t*, S(t) = 1 means that the maximum number of motor units that can be activated at the same time are activated at time *t*.  $t_0$  is a time shift that can be derived by the equilibrium condition that the muscles have to be activated at the beginning of the movement to hold the mass (i.e. exert a certain force). The force  $f_m$  of the muscle is modelled by  $f_m = S(t)f$ .

Finally, the relationship between the muscle force  $f_m$  and the external force F can be calculated by a geometry function G(X) depending on the distance X between hip and ankle,  $F = f_m G(X)$  [8]. To formulate G individually for the knee joint, anthropometric data like the moment arm of the model muscle (estimated by the radius of the knee joint), the length of thigh and shank and the distance between the patella centre and the tuberositas tibiae are needed. For more details for the measurement of these quantities, see [9].

We get the following model equations (cf. [12]):

$$m\ddot{X} = -mg\sin\alpha + S(t)G(t)\left(\frac{c}{G(X)\ddot{X} + b} - a\right),\tag{12}$$

$$G(X) = \frac{r\sin\beta}{l_0 l_u \sin\sigma} X,$$
(13)

$$\sigma = 2\beta + \arcsin\left(\frac{r}{k_{\rm o}}\sin\beta\right) + \arcsin\left(\frac{r}{k_{\rm u}}\sin\beta\right),\tag{14}$$

$$X = \sqrt{l_o^2 + l_u^2 - 2l_0 l_u \cos \sigma},$$
(15)

$$S(t) = 1 - \exp(-A(t - t_0) + 1 - \exp(-A(t - t_0))),$$
(16)

$$S(t_0) = \frac{G(X_0)f_{\text{max}}}{mg\sin\alpha}.$$
(17)

 $\sigma$  denotes the knee angle and  $\beta$  is the angle between muscle and knee.  $l_0$  is the length of the thigh,  $l_u$  the length of the shank,  $k_0$ ,  $k_u$  the position of the muscle and r the knee radius.

**Remark 2:** The anthropometric parameters can be measured directly, whereas the neuromuscular parameters a, b, c (Hill's force–velocity relation), and A (describing the activation) have to be identified. In the model equations the mass m, the gravitational acceleration g, the inclination angle  $\alpha$ , the initial position  $X_0$  and the initial velocity  $V_0$  are conditions of

the movement, all other parameters are properties of the subject. Equation (17) is the equilibrium condition determining  $t_0$  in the case that the initial velocity  $V_0$  is zero.

## 4. Measurements and parameter estimation

To show the individual differences in the property values, we performed measurements and identified the parameters of the model equations that were not measured directly. The validity, reliability and objectivity of the method was investigated in detail before (cf., for example [4,12,15]), where also statistical analysis of measurements and identified parameters were presented.

#### 4.1. Measurements

To determine the parameters of the extension model individually, we measured kinematics and kinetics of real leg extension movements executed by our subjects. The measurements complied with the requirements of the local university, as well as current local law and regulations. Written informed consent was obtained from the athletes prior to any testing. The used measurement device is an inclined leg press (Tetra<sup>®</sup> Illmenau) with a force platform (Kistler<sup>®</sup>) on a sledge. This sledge can be fixated to measure isometric movements and can be freely moved to measure concentric leg extensions with different loads and inclination angles. The different inclinations were used to simulate different movement conditions at a wide range of contraction velocities. Besides the force exerted on the sledge we measured position and velocity of the sledge at 500 Hz. Furthermore, we measured movement conditions (e.g. moved load) and anthropometrical properties of the subject (e.g. thigh length) directly.

# 4.2. Subjects

Our subjects were 10 sports students (6 male, 4 female, age  $23.5 \pm 0.9$  years, height  $1.73 \pm 0.1$  m, weight  $67.9 \pm 9.15$  kg). They performed two test series, each including two isometric and four dynamic concentric movements with two different inclination angles  $(14^{\circ}, 28^{\circ})$  and pushed mass 45 kg on an inclined leg press.

**Remark 1:** The aim of our measurements was not to show statistical correlations but to reveal individual differences. Data on larger groups of subjects can be found in [4].

#### 4.3. Parameter estimation

All parameters which could not be measured directly were determined by non-linear parameter estimation with a custom-made software (JOP kinematics) based on a modified Levenberg–Marquart algorithm. Briefly, the measured kinematics and kinetics of the movement were compared with the data of the simulated movement. Then, the model parameters were altered until simulation and real movement coincide sufficiently. For details see [12]. Thus, the determined parameters describe the muscle force–contraction velocity relationship as well as the activation rate of the muscle. The individual parameters  $f_{\text{max}}$  [N] (isometric force in the muscle),  $p_{\text{max}}$  [W] (maximum possible power of the muscle) and  $v_{\text{max}}$  [m/s] (maximum possible contraction velocity) define the Hill-type extensor model muscle. The activation parameter A [1/s] describes the rate at which muscle fibres are activated.

# 4.4. Results of measurements

The parameters  $f_{\text{max}}$ ,  $v_{\text{max}}$  and  $p_{\text{max}}$  in Hill's equation and the activation parameter A were identified. The mean values and the standard deviations of the identified parameters were  $f_{\text{max}} = 11111 \pm 3264$  N,  $v_{\text{max}} = 1.29 \pm 0.63$  m/s,  $p_{\text{max}} = 1173 \pm 324$  W,  $A = 12.0 \pm 3.0 \text{ s}^{-1}$ . Significant differences (*t*-test, p < 0.05) between male and female subjects could be observed for the values of isometric muscle force  $f_{\text{max}}$  (male: 13178  $\pm$  1935 N, female: 8011  $\pm$  2090 N) and the maximum power  $p_{\text{max}}$  (male: 1361  $\pm$  190 W, female: 891  $\pm$  280 W), whereas the parameter values of the maximum velocity and the activation do not differ significantly (see, for example [4]).

**Remark:** Note the difference between  $f_{\text{max}}$ , the isometric force of the muscle at maximum activation and max F(t), the maximum of the time-dependent force during the movement, measured on the force platform. The parameter  $f_{\text{max}}$  is a property of the subject and therefore independent of the specific movement, whereas max F(t) crucially depends on the movement conditions and the person's properties, including the activation function and the anthropometric quantities.

The shape of Hill's force–velocity relation differs substantially between subjects, as can be seen in Figure 1. All cases of intersection as described in Section 2.2 occur in the experimental data. The left diagram in Figure 1 shows subject #4 (black line, male) and subject #9 (grey line, female), the diagram in the middle shows subject #3 (black line, male) and subject #9 (grey line, female) and the diagram on the right side shows subject #2 (black line, male) and subject #6 (grey line, male).

The maximum of the velocity (max V(t)) of the pushed mass and the maximum force (max F(t)) measured on the force platform clearly depend on the inclination angle. As expected, the mean value of the maximum velocity of the pushed mass is larger for an inclination angle of 14° than the maximum velocity occurring at inclination angle of 28° (p < 0.1). For the maximum force measured at the force plate, we have a larger mean value for the greater inclination angle (p < 0.1). The measured force maxima at small angle correlate significantly with the force maxima at larger angle (r = 0.955) and there is also a significant correlation between the maxima of the velocities (r = 0.962).

Tables 1 and 2 show the ranking of the 10 subjects for inclination angles of  $14^{\circ}$  and  $28^{\circ}$ . Concerning force, a large inclination angle is advantageous for subject #3 who is second best in achieving a high force on the platform at  $28^{\circ}$ , whereas only on fifth position at  $14^{\circ}$ .

|            | ( )    |        |        | -      |        | .,     |          | -      |        |        |  |
|------------|--------|--------|--------|--------|--------|--------|----------|--------|--------|--------|--|
| Subject    | 1      | 2      | 3      | 4      | 5      | 6      | 7        | 8      | 9      | 10     |  |
| 14°<br>28° | 3<br>4 | 7<br>6 | 5<br>2 | 2<br>5 | 6<br>8 | 1<br>1 | 10<br>10 | 9<br>9 | 4<br>3 | 8<br>7 |  |

Table 1. max F(t): Ranking of the subjects in max F(t) at different inclination angles.

| Table 2. | $\max V(t)$ : | Ranking | of the subj | jects in max | V(t) | at different | inclination | angles. |
|----------|---------------|---------|-------------|--------------|------|--------------|-------------|---------|
|          |               |         |             |              |      |              |             |         |

|         |   |   |   |   |   |   |   |    |   | -  |  |  |
|---------|---|---|---|---|---|---|---|----|---|----|--|--|
| Subject | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8  | 9 | 10 |  |  |
| 14°     | 4 | 1 | 5 | 2 | 3 | 8 | 6 | 10 | 7 | 9  |  |  |
| 28°     | 5 | 2 | 3 | 1 | 4 | 6 | 7 | 10 | 8 | 9  |  |  |

#### 5. Simulations

#### 5.1. Interindividual differences

To show some effects of the interindividual differences in the force–velocity relation on the sports performance, we simulate the muscle force at constant contraction velocities 0, 0.2, 0.4 and 0.6 m/s. Larger velocities would exceed  $v_{\text{max}}$  for some subjects. In particular, we are interested in the ranking of the subjects and in the differences in the forces between men (subjects #1–6) and women (subjects #7–10). Using the identified parameters  $f_{\text{max}}$ ,  $v_{\text{max}}$  and  $p_{\text{max}}$  for the 10 subjects, we evaluate the force–velocity relations. The results are collected in Table 3. For the muscle force at velocity 0.6 m/s, we get a mean value of  $1426 \pm 749$  N. There is no significant difference between male and female in the value of f(0.6).

Subjects #1 and #3 (both male) have very low forces at velocity 0.6 m/s, lower than all female subjects (#7 – #10). This is due to their low values of  $v_{\text{max}}$ . Table 4 shows the ranking of the 10 subjects in  $f_{\text{max}}$ , f(0.2), f(0.4) and f(0.6).

The ranking reflects the different shapes of the force–velocity relation. Subjects with low  $v_{max}$  exert lower force at high contraction velocity. Note that the ranking may change within different contraction velocities. Compared with the other subjects, subject #6 has mean force at low and high velocities, whereas in the medium range of velocity the force is high. This is due to a low curvature of the force–velocity relation (cf. Figure 1).

**Remark 1:** To simulate forces f(0.2), f(0.4) and f(0.6), we had to use all muscle parameters of Hill's force–velocity relation. Thus, we see that the muscle force at a certain velocity does not only depend on the isometric force  $f_{\text{max}}$  but also on the maximum contraction velocity and the maximum power.

**Remark 2:** We have seen that a measurement of the muscle force at a certain velocity does not give information on the force at another contraction velocity. This fact can be generalized for all changes in conditions. If a muscle test does not measure properties, the result depends on the conditions of the measurement (in this case the contraction velocity) and on other properties of the subject and therefore cannot be used for movements under different conditions.

Table 3. Muscle forces at contraction velocities 0, 0.2, 0.4 and 0.6 m/s.

| Subject   | 1     | 2     | 3     | 4     | 5     | 6     | 7    | 8     | 9    | 10   |
|---|-------|-------|-------|-------|-------|-------|------|-------|------|------|
| $ \frac{f_{\text{max}}[N]}{f(0.2)[N]} \\ f(0.4)[N] \\ f(0.6)[N] $ | 12256 | 13915 | 14242 | 16029 | 10572 | 12051 | 6178 | 10947 | 7970 | 6950 |
|   | 6097  | 5433  | 5870  | 7084  | 5267  | 6915  | 3017 | 4048  | 5070 | 3145 |
|   | 2726  | 3141  | 2291  | 4040  | 3092  | 3912  | 1651 | 1868  | 3239 | 1888 |
|   | 600   | 2075  | 303   | 2505  | 1909  | 1941  | 888  | 798   | 1978 | 1262 |

Table 4. Ranking in the muscle forces at contraction velocities 0, 0.2, 0.4 and 0.6 m/s.

| Subject                | 1 | 2      | 3      | 4      | 5      | 6      | 7        | 8      | 9      | 10     |
|------------------------|---|--------|--------|--------|--------|--------|----------|--------|--------|--------|
| $f_{\max}[N]$          | 4 | 3      | 2      | 1      | 7      | 5      | 10       | 6      | 8      | 9      |
| f(0.2)[N]<br>f(0.4)[N] | 3 | 5<br>4 | 4<br>7 | 1<br>1 | 6<br>5 | 2<br>2 | 10<br>10 | 8<br>9 | 7<br>3 | 9<br>8 |
| f(0.6)[N]              | 9 | 2      | 10     | 1      | 5      | 4      | 7        | 8      | 3      | 6      |

Simulations of max F(t) and max V(t) using the model equations show similar results as the measurements and can elucidate the causes for changes in the ranking as described in Tables 1 and 2. Since the movement at a smaller inclination angle is faster and the activation at the beginning is less (see Equation (17) of the equilibrium condition), subjects with higher activation parameter have an advantage at this movement. A similar discussion can be found in [16], where a jump on the moon is discussed. Less gravitation implies that a vertically accelerated mass has less weight but the same inertia, a situation analogous to a pushed mass on an inclined leg press.

# 5.2. Visualization of the performance function

For the following simulations we define the maximum velocity of the pushed mass, max V(t), to be the performance under consideration. The velocity V is the derivative of the solution X of the model Equations (12)–(17). So for fixed conditions  $m, g, X_0, V_0$  and  $\alpha$ , the performance function f (Definition 2.2) maps the person's state  $x = v_{\text{max}}, f_{\text{max}}, P_{\text{max}}$ A,  $l_0, l_u k_0, k_u, r$ ) to the associated performance  $z = f(x) = \max(dX/dt)$ .

For visualizing the performance function, we keep all the parameters except  $v_{\text{max}}$  and  $f_{\text{max}}$  fixed and consider the restriction of *f* to the subspace  $R^2$ . We let the isometric force  $f_{\text{max}}$  vary between 7000 and 15000 N, the maximum contraction velocity between 0.6 and 1.5 m/s. First, we simulated the performance function for the mean values of the measured or identified parameters ( $p_{\text{max}}$ , A,  $l_o$ ,  $l_u k_o$ ,  $k_u$ , r) = (1173 W, 12 s<sup>-1</sup>, 0.42 m, 0.4 m, 0.42 m, 0.07 m, 0.06 m). For the conditions, we took m = 45 kg,  $g = 9.81 \text{ m/s}^{-2}$  and  $V_0 = 0$ . For the



Figure 2. Performance function: parameters  $v_{\text{max}}$  and  $f_{\text{max}}$  are mapped to the performance max V(t) with inclination angles 14°, 28°, 60° and 90° from top left to bottom right. All other parameters are kept constant.

initial position  $X_0 = 0.5$  m, we investigated the effects of the inclination angles  $14^\circ$ ,  $28^\circ$ ,  $60^\circ$  and  $90^\circ$  (see Figure 2). Then we kept the inclination angle  $\alpha$  constant at  $28^\circ$  and set the initial position  $X_0$  at 0.35, 0.45, 0.55 and 0.70 m (see Figure 3).

In Figure 2 we can see performance limiting factors: at an inclination of  $14^\circ$ , an increase in the parameter  $f_{\text{max}}$ , the isometric force above 1000 N does not lead to a better performance for low contraction velocities  $v_{\text{max}}$  (top left). Note that  $f_{\text{max}}$  is a performance limiting factor in the sense of Definition 2.2 only if we restrict the performance function on the 2D domain ( $v_{\text{max}}, f_{\text{max}}$ ). The simulation result is according to our experience: the movement at low inclination is faster. If the maximum contraction velocity of a person is too low, an increase of force does not enhance the performance. A similar situation is obtained for  $v_{\text{max}}$  at an inclination angle of 90° (bottom right): larger  $v_{\text{max}}$  does not increase the performance, if the isometric force is low. In Figure 2 (bottom left, angle 60°), the parameter  $v_{\text{max}}$  is a performance determining factor at every state.

**Remark 3:** Note that the parameter used in the simulations for Figures 2 and 3 are mean values and do not describe one of the subjects. As the subjects differ substantially in all parameter values, it is not possible to see the results of the measurements in these graphics.

To show the individual differences in the performance and the effect of changes in the parameters  $v_{\text{max}}$  and  $f_{\text{max}}$ , we simulated two subjects with their individual parameters (see Figure 4).



Figure 3. Parameters  $v_{\text{max}}$  and  $f_{\text{max}}$  are mapped to the performance max V(t), from top left to bottom right:  $X_0 = 0.35, 0.45, 0.55$  and 0.7 m. All other parameters are kept constant.



Figure 4.  $X_0 = 0.45$  m,  $V_0 = 0$  m/s,  $\alpha = 14^\circ$ . Left: subject #3, the state is at ( $v_{max} = 0.641$  m/s,  $f_{max} = 14242$  N). Right: subject #9, the state is at ( $v_{max} = 1.123$  m/s,  $f_{max} = 7970$  N).

**Remark 4:** If the changes in  $f_{\text{max}}$  are achieved by training, it is not possible to keep the maximum power  $p_{\text{max}}$  constant. Training always leads to changes in all parameters describing the force–velocity relation [17]. To get a complete visualization of the effects of changes in parameter values, one has to look at series of graphs at different values for the maximum power and the activation (cf. [18]).

## 6. Conclusions

Individual measurements confirm that the variation in neuromuscular properties between different subjects is substantial. To get reliable results from subject-specific simulations, the individual determination of these properties is of great importance. The performance function of a specific movement provides information about the necessary property change that would lead to the largest increase in performance. Therefore, simulation using subject-specific neuromuscular properties is a promising method for planning and controlling of training.

#### References

- V.M. Zatsiorsky, Science and Practise of Strength Training, Human Kinetics, Champaign, IL, 1995.
- [2] L.L. Menegaldo and L. Fernandes de Oliveira, Effect of muscle model parameter scaling for isometric plantar flexion torque prediction. J. Biom. 42 (15) (2009), pp. 2597–2601.
- [3] C.R. Winby, D.G. Lloyd, and T.B. Kirk, Evaluation of different analytical methods for subjectspecific scaling of musculotendon parameters. J. Biom. 41 (8) (2008), pp. 1682–1688.
- [4] S. Thaller and H. Wagner, *The relation between Hill's equation and individual muscle properties*. J. Theor. Biol. 231 (2004), pp. 319–332.
- [5] C.Y. Scovil and J. L. Ronsky, Sensitivity of a Hill-based muscle model to perturbations in model parameters. J. Biom. 39 (11) (2006), pp. 2055–2063.
- [6] D.G. Thelen, Adjustment of muscle mechanics model parameters to simulate dynamic contractions in older adults. J. Biom. Eng. 125 (2003), pp. 70–77.
- [7] A. Nagano and K.G.M. Gerritsen, *Effects of neuromuscular strength training on vertical jumping performance: A computer simulation study.* J. Appl. Biom. 17 (2001), pp. 113–128.
- [8] M. Sust, Biomechanische Aspekte der definition von Maximal- und Schnellkraft. Theorie und Praxis der Koerperkultur 3612 (1978), pp. 763–768.
- [9] M. Sust, T. Schmalz, and S. Linnenbecker, *Relationship between distribution of muscle fibres and invariables of motion*. Human Mov. Sci. 16 (1997), pp. 533–546.

- [10] J.P. Zandwijk, G.C. Baan, M.F. Bobbert, and P.A. Huijing, *Evaluation of a self-consistent method for calculation of muscle parameters from a set of isokinetic releases*. Biol. Cybernetics 77 (1997), pp. 277–281.
- [11] T. Siebert, H. Wagner, and R. Blickhan, Not all oscillations are rubbish: Forward simulation of quick-release experiments. J. Mec. Med. Biol. 3 (2003), pp. 1–16.
- [12] T. Siebert, M. Sust, S. Thaller, M. Tilp, and H. Wagner, An improved method to determine neuromuscular properties using force laws-from single muscle to applications in human movements. Human Mov. Sci. 26 (2007), pp. 320–341.
- [13] M. Sust, Hillsche Gleichung aus theoretischer Sicht. DVS-Protokolle 55 (1993), pp. 126–136.
- [14] A.V. Hill, *First and Last Experiments in Muscle Mechanics*, Cambridge, Cambridge University Press, New York, 1970.
- [15] H. Wagner, S. Thaller, R. Dahse, and M. Sust, *Biomechanical muscle properties and angiotensinconverting enzyme gene polymorphism: A model-based study.* Eur. J. Appl. Physiol. 98 (5) (2006), pp. 507–515.
- [16] S. Thaller and M. Sust, Die Bedeutung der Muskeleigenschaften in unterschiedlichen Gravitationsfeldern, Spectrum d. Sportwiss. 15 (2) (2003), pp. 60–72.
- [17] M. Tamme, Die Veraenderung muskelspezifischer Parameter durch Training mit hohen Lasten, Schriften der DVS 55 (1993), pp. 137–141.
- [18] S. Thaller, M. Sust, and M. Tilp, *Determination of individual neuromuscular properties and applications in sports science*, Proceedings MATHMOD 09 Vienna, ARGESIM Reports 35, 2009, pp. 1040–1045.

Copyright of Mathematical & Computer Modelling of Dynamical Systems is the property of Taylor & Francis Ltd and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.